THE EQUATIONS OF KINEMATIC REFLECTION

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Abstract

In this paper we show that the correct application of Huygen`s Principle at the surface of a mirror which is in movement with respect to the light propagation medium has led us to modify the classical laws of reflection, which were established for static situations. Because in practice all optical elements are in movement together with the Earth around the Sun at 30 km/sec, in reality, we are actually always in a kinematic situation, and for very accurate observations we must consider the laws of this new optics, Kinematic Optics.

Introduction

The classical laws of optics were established on the assumption that the light sources, the medium of propagation of light, and the optical elements such as mirrors, lenses, prisms etc. are fixed. But in reality all these are in movement along with the Earth around the Sun. Because this we are obliged consider a new optics: “kinematic optics”.

In this article we tackle the problems of kinematic reflection.

To understand this, we will treat in parallel aspects of classical reflection which we have named static reflection, in comparison with this new reflection, which we have named “kinematic reflection“.

The reasoning assumes that the light waves and the optical elements are in movement through the ether, the light transmission medium, which is considered to be fixed.

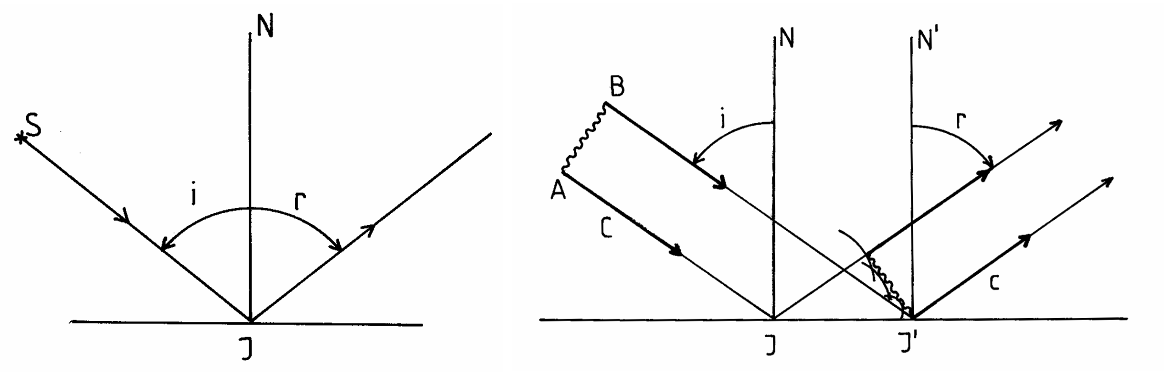
1. *Two-dimensional static reflection*

The fact is well known that the following two laws of geometrical optics have been established in accordance with experimental results:

1st Law: *The incident beam, the perpendicular to the reflective surface, and the reflected beam lie in the same plane. See Fig.1 a).*

2nd Law :*The angle of reflection r and the angle i made by the incident beam and the perpendicular* ***N*** *to the reflective surface at the point of incidence are equal:* $i = r$.

On the other hand light reflection can be explained using the Huygens-Fresnel principle in the framework of wave optics theory. See *Fig.1 b).*



a) b)

Fig.1** Classical representation of the reflection **a)** and the explanation of it using Huygens- Fresnel principle **b)

***Observation**:*

*The statements above are valid only in the case of a static situation when the light source **S**, the reflective surface **M** and the medium through which the light wave is propagating are static (motionless) with respect to one another (i.e. in a situation of relative rest).*

Because we have been considering a plane wave, we term this kind of reflection ***static two-dimensional reflection.***

2. Considerations of kinematic optics

In practice, situations are encountered when the relative speed of optical devices with respect to a light source has to be taken into account. An example of such a situation is the explanation of astronomical light aberration phenomena. It is well known from astronomy that, for the image of a star S to appear in the reticule of a telescope, the telescope needs to be inclined at an angle of ***σ*** in order for the speed of movement of the Earth to be compensated, as shown in ***Fig. 2 a)***

The aberration angle is given by the formula:

$$\sin\sigma = \frac{v}{c}\sin\theta$$

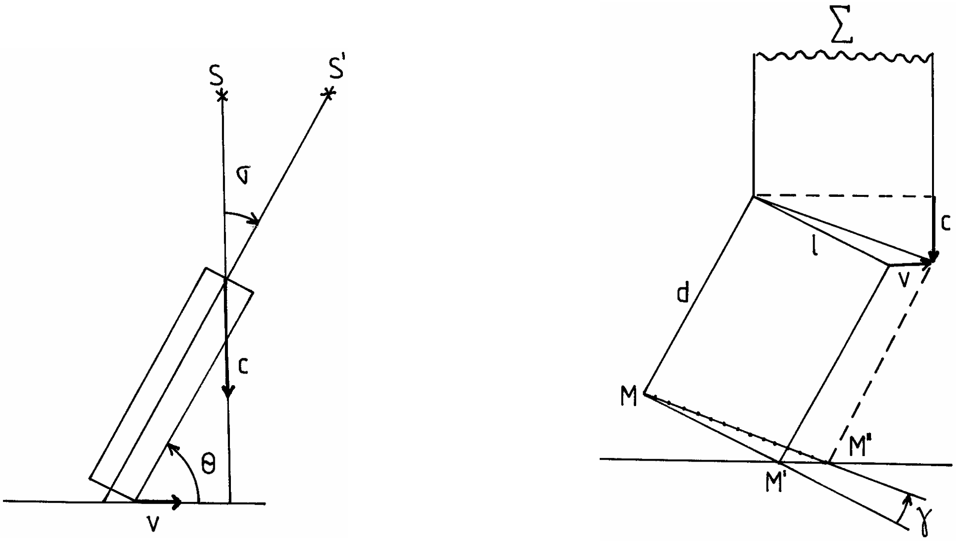
Where: $v = 30Km \cdot s^{-1}$ is the speed of movement of the Earth around the Sun on its orbit and $c = 300000Km \cdot s^{-1}$ is the speed of light.

Observation:

The formula above has been arrived at by taking into account just one single beam of light.

If the Huygens-Fresnel principle is used to explain the astronomical aberration of light, the situation gets more complicated.

Let us consider a reflecting telescope of ***d*** length and aperture ***l.*** The wave surface ***Σ*** coming from the star ***S*** will successively arrive at points ***MM'*** on the surface of the mirror, which is moving at the speed *v* together with the Earth, in points on a virtual surface ***MM''***. This virtual surface ***MM''*** makes an angle ***γ*** with the actual surface of the mirror ***MM'***. See ***Fig.2 b)***



a) b)

Fig. 2 *Astronomical light aberration phenomena* ***a)***
and the explanation using Huygens- Fresnel principle ***b)***

Observation:

If this point of view is not accepted, then one needs to return to the antique concept of ***instantaneous propagation of light*** *(action at a distance). This example forces us to reconsider the phenomena of light reflection in kinematic situations.*

3. *Two-dimensional kinematic reflection*

Let us consider a kinematic situation in a light-transmitting medium when a mirror ***MM'*** is moving with respect to the medium and is receding from the light source. See ***Fig. 3.***

The first contact between the wave front Σ and the mirror will take place at point ***A'*** and the last one at point ***B''***. The line ***A'B'',*** which is a virtual reflective surface, makes the angle γ with the mirror ***MM*** .

Putting the equations $B'B''=ct$; $M'B''=v\cdot t$ and using the simple relations expressed by the triangles ***A'B'B''*** and ***A'B'M',*** the follow formula for γ angle can be obtained:

$$tg\gamma = \frac{v\sin i \cdot \sin\alpha}{c \pm v\cos i \cdot \sin\alpha}$$ where:

c is the speed of light in the medium;

v is the speed of the mirror with respect to the medium;

i is the angle of incidence;

α is the angle between the surface of the mirror and the velocity ***v.***

The sign in the above equation is negative when the mirror is receding from the wave front and positive when the mirror is approaching the wave front.

The situation is as though the mirror ***MM*** has experienced a rotation through the angle γ.

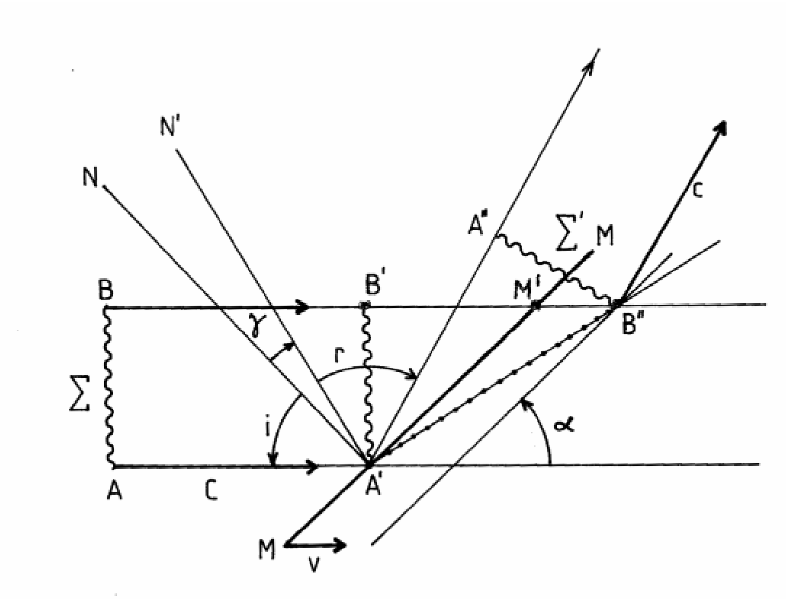


Fig.3 *The use of the Huygens-Fresnel principle in the case of a mirror that moves with respect to the medium through which the light travels.*
Bi-dimensional kinematic reflection

The envelope of the elementary waves emanating from centres of oscillation along the line ***A'B''*** is given by the reflected wave front ***Σ'***.

The reflection takes place with respect the perpendicular ***N'*** to the surface ***A'B''*** which makes an angle ***γ*** with the perpendicular ***N*** to the surface ***MM***. As a result, the angle of incidence ***i'*** with respect to the perpendicular ***N'*** will have the value $i' = i \pm \gamma$. In this situation, the **2nd** law of reflection in the case of a two-dimensional kinematic situation becomes $r = i \pm \gamma$.

For example : if $i = \alpha = 45^o$, $v = 30 Km \cdot s^{-1}$ and $c = 300000 Km \cdot s^{-1}$, the angle γ has the value 10.33 ″. This can not be neglected in the astronomical determinations.

Because a plane wave has been used in our reasoning, we call this kind of reflection ***2-dimensional kinematic reflection***.

Observation:

The first law of reflection remains unchanged.

4. *Three-dimension static reflection*

Let us consider a bundle of parallel light rays in which the wave front ***OAB*** has the shape of a right angled triangle parallel with the plane ***XO'Y*** of an ***O'XYZ*** coordinate system. We consider a mirror ***M*** placed at the origin of the system, in a position such that the projection of the wave front ***OAB*** describes the triangle ***O'A'B'*** on it.

From ***Fig.4*** it can be seen that intersection of the mirror with the plane ***ZO'X*** is given by the line ***O'A'*** which makes the angle β with the ***O'X*** axis, and the intersection of the mirror with the ***ZO'Y*** plane is given by the line ***O'B*** which makes the angle ***θ*** with the ***O'Y*** axis.

We consider three perpendiculars ***N***, N_A and N_B respectively at the points ***O'***, ***A'*** and ***B'*** that are mutually parallel. These three perpendiculars determine a bundle of perpendiculars, with respect to which the rays of light that are parallel with the ***O'Z*** axis are reflected.

After reflection we will get a bundle of parallel rays with the wave front ***O''A"B"***. For each ray of light the classical laws of reflection are valid separately.

To find the direction of the perpendicular ***N*** in space, one has to write the direction cosine equations with respect to the ***X, Y*** and ***Z*** axis.

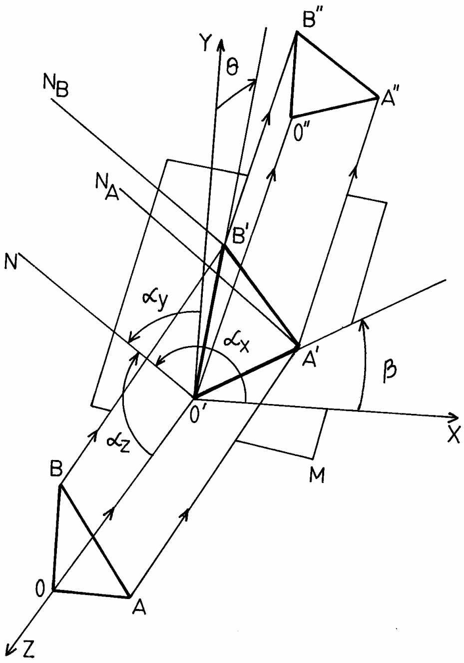


Fig.4 *Illustrating the static reflection of a bundle of parallel light rays*

To do this we start from the equations determining the plane ***O'A'B'***, and take into account the coordinates of the points ***O', A'*** and ***B' O'(0, 0, 0,) ; A' (a, 0, a·tgβ), B' (0, b, b·tg θ***), where OA =a , OB =b.

From the equations: $X\cos\alpha_x + Y\cos\alpha_y + Z\cos\alpha_z = 0$

$$\cos^2\alpha_x + \cos^2\alpha_y + \cos^2\alpha_z = 1$$

We obtain for the direction cosines the following formulas:

$$\cos\alpha_x = tg\beta \Big/ \sqrt{\left(tg^2\beta + tg^2\theta + 1\right)}$$

$$\cos\alpha_x = tg\beta \Big/ \sqrt{\left(tg^2\beta + tg^2\theta + 1\right)}$$

$$\cos\alpha_z = 1 \Big/ \sqrt{\left(tg^2\beta + tg^2\theta + 1\right)}$$

5. *Three-dimensional kinematic reflection*

Let us now suppose that the mirror ***M*** has a movement of translation with respect to the medium along the positive direction of the ***OZ*** axis, with the velocity ***v***.

From ***Fig.5*** we see that:

The first contact of the wave front with the mirror ***M*** takes place at the point ***O'***;

In the ***XO'Z*** plane, the wave front will catch up the mirror at the point along the ***O'A"*** line;

In the ***ZO'Y*** plane, the wave front will meet the mirror ***M*** at the points along the ***O'B"*** line; and

The reflection of the waves will take place from the virtual surface of the triangle ***O'A"B".*** The perpendicular ***N"*** to this is no longer identical with the perpendicular ***N'*** to the mirror ***M.***

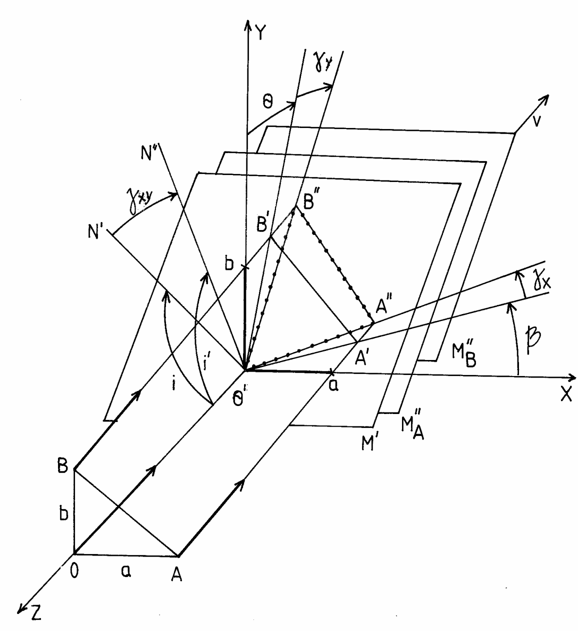


Fig.5 *The representation of the virtual reflection surface(O'A"B") of a mirror (O'A'B') that moves with the constant speed v*

The direction in space of this perpendicular ***N"*** can be found by writing the direction cosines with respect to the ***X,Y*** and ***Z*** axes. To do this, we start with the equation of the ***O'A"B"*** plane, and consider the coordinates of the points ***O',A"*** and ***B":*** ***O' (0,0,0) ; A''(a, 0,atg (***$\beta+\gamma_x$***)) ; B''(0, b, b tg(*** $\theta+\gamma_y$***)).***

This time the angles β and θ are increased by the values γ_x and γ_y respectively, because of the movement of the mirror and because of the occurrence of kinematic reflection.

In a similar way the direction cosines of the perpendicular ***O'N"*** have the following values:

$$\cos\alpha_x'' = tg(\beta+\gamma_x)\Big/\sqrt{tg^2(\beta+\gamma_x)+tg^2(\theta+\gamma_y)+1}$$

$$\cos\alpha_y'' = tg(\theta+\gamma_y)\Big/\sqrt{tg^2(\beta+\gamma_x)+tg^2(\theta+\gamma_y)+1}$$

$$\cos\alpha_z'' = 1\Big/\sqrt{tg^2(\beta+\gamma_x)+tg^2(\theta+\gamma_y)+1}$$

The values of the angles γ_x and γ_y are determined using the following formula

$$tg\gamma = v\sin i\sin\alpha/(c \pm v\cos i\sin\alpha)$$

And, considering the two-dimensional reflection at the plane ***ZO'X*** for γ_x and at the plane ***ZO'Y*** for γ_x respectively, as can be seen from ***Fig. 6***.

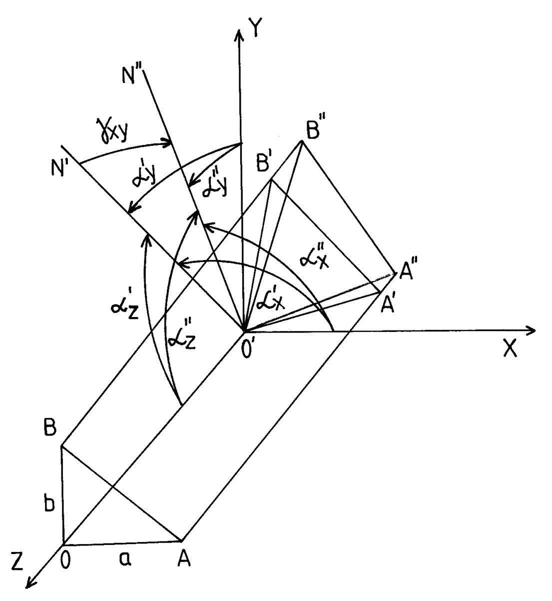


Fig.6 *Representation of the perpendicular N' on the mirror O'A'B' and of the virtual reflection surface with respect to the O'XYZ coordinates system*

Starting with the following equations: $Xtg\beta + Ytg\theta - Z = 0$

$$Xtg(\beta + \gamma_x) + Ytg(\theta + \gamma_y) - Z = 0$$

And using the formula: $\cos\varphi = (A_1A_2 + B_1B_2 + C_1C_2) / \sqrt{(A_1^2 + B_1^2 + C_1^2)\cdot(A_2^2 + B_2^2 + C_2^2)}$

Where the coefficients have the following values:

$$A_1 = tg\beta;\ B_1 = tg\theta;\ C_1 = -1;\ A_2 = tg(\beta + \gamma_x);\ B_2 = tg(\theta + \gamma_y);\ C_2 = -1$$

we get the angle γ_{xy} between the plane of the mirror ***O'A'B'*** and the plane ***O'A"B"*** on which the elementary oscillating sources are placed during the reflection process.

This angle can be computed using the following formula:

$$\cos\gamma_{xy} = \frac{tg\beta\cdot tg(\beta + \gamma_x) + tg\theta\cdot tg(\theta + \gamma_y) + 1}{\sqrt{(tg^2\beta + tg^2\theta + 1)\cdot(tg^2(\beta + \gamma_x)\cdot tg^2(\theta + \gamma_y) + 1)}}$$

In this case the angle of reflection ***r"*** will have the following value: $r'' = i + \gamma_{xy}$.

This expression represents the second law of three-dimensional kinematic reflection.

In a similar way, starting with the following equations:

$$X\cos\alpha_y - Y\cos\alpha_x = 0$$

$$X\cos\alpha_y^{'} - Y\cos\alpha_x^{1} = 0$$

Where the coefficients have the following values:

$A_1 = \cos\alpha_y;\ B_1 = -\cos\alpha_x;\ C_1 = 0$ and $A_2 = \cos\alpha_y^{'};\ B_2 = -\cos\alpha_x^{'};\ C_2 = 0$

We get the value for the angle γ_{ir} between the plane ***ZON'*** containing the perpendicular ***N'*** to the mirror ***M*** and the plane ***ZO'N"*** containing the perpendicular ***N"*** to the surface ***O'A"B"*** using the following formula:

$$\cos\gamma_{ir} = \frac{tg\theta \cdot tg(\theta+\gamma_y)+tg\beta \cdot tg(\beta+\gamma_x)}{\sqrt{(tg^2\theta+tg^2\beta)\cdot(tg^2(\beta+\gamma_x)+tg^2(\theta+\gamma_y))}}$$

The fact that an angle appears between the plane ***ZO'N'*** that contains the incident beam and the plane ***ZO'N"*** that contains the reflected beam means that, (see ***Fig.*** *7*), in the case of three-dimensional kinematical reflection, the **1st** law of reflection is no longer valid in the form in which it has been accepted up till now. The appearance of the angle γ_{xy} proves the change of the 2nd law of reflection in the case of kinematic reflection.

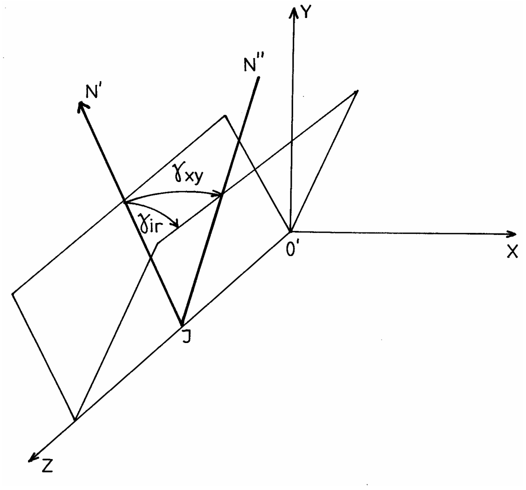


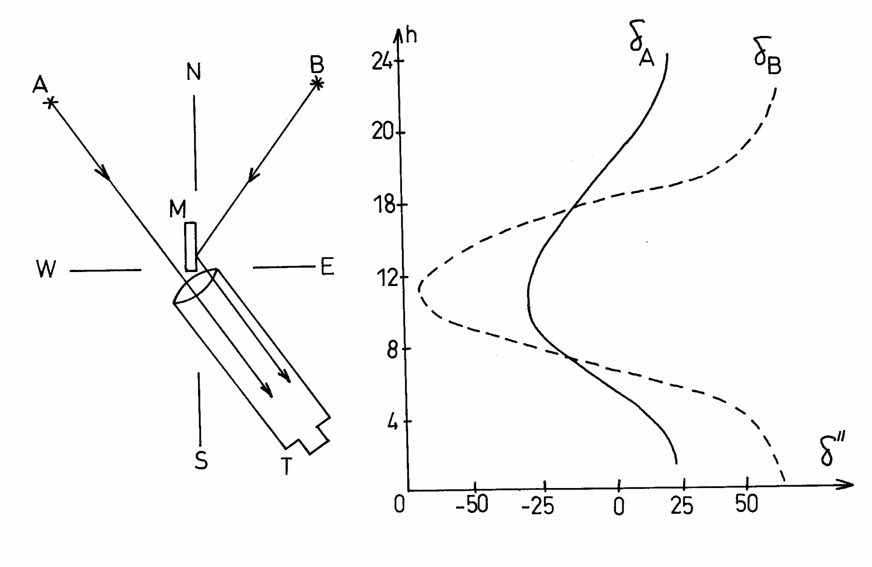
Fig.7 *The representation of the angle γ_{xy} made by the perpendicular* ***N'*** *on the moving mirror and the perpendicular* ***N''*** *on the virtual reflective surface, and the representation of the angle γ_{ir} made between the planes containing the perpendiculars N' and N".*

6. Preliminary experimental results

In order to verify whether the kinematic reflection exists, we made an original light deviation experiment with a fixed terrestrial telescope T . In front of the telescope a mirror M was placed at half of the objective O, see ***Fig.8 a)***. So we were able to see directly the image of a mark A , and also the image of a mark B after reflection from the mirror.

The directions of Mark A and mark B made an angle of ±35° with the direction N of north.

The determinations were made for two weeks, day and night. Each hour the positions of the marks were read with the precision of 1" arcsecond. Due to movement of the telescope with the Earth around the Sun, the deviation of the image of the mark B after reflection was twice the deviation of mark A. See ***Fig. 8 b)***. We consider that this difference in deviation can be explained only by kinematic reflection.



a) b)

Fig.8 *The experimental set-up used to verify the appearance of kinematic reflection* ***a)*** *and deviations in time of the images of two marks considered in the experiment* ***b)***

Acknowledgements

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